Georgia
Standards of Excellence
Course Curriculum
Overview

Mathematics

Accelerated GSE Pre-Calculus

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

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Accelerated GSE Pre-Calculus Curriculum Map

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These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.
All units will include the Mathematical Practices and indicate skills to maintain.

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.
Number and Quantity Strand: RN = The Real Number System, Q = Quantities, CN = Complex Number System, VM = Vector and Matrix Quantities
Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities
Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions
Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry
Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions

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The Comprehensive Course Overviews are designed to provide access to multiple sources of support for implementing and instructing courses involving the Georgia Standards of Excellence.

Accelerated GSE Pre-Calculus

Accelerated Pre-Calculus is the third in a sequence of mathematics courses designed to ensure that students are prepared to take higher-level mathematics courses during their high school career, including Advanced Placement Calculus AB, Advanced Placement Calculus BC, and Advanced Placement Statistics.

The standards in the three-course high school sequence specify the mathematics that all students should study in order to be college and career ready. Additional mathematics content is provided in fourth credit courses and advanced courses including, calculus, advanced statistics, discrete mathematics, and mathematics of finance courses. High school course content standards are listed by conceptual categories including Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Conceptual categories portray a coherent view of high school mathematics content; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Standards for Mathematical Practice provide the foundation for instruction and assessment.

GSE Pre-Calculus: Unit Descriptions

Accelerated Pre-Calculus focuses on standards to prepare students for a more intense study of mathematics. The critical areas organized in nine units delve deeper into content from previous courses. The study of circles and parabolas is extended to include other conics such as ellipses and hyperbolas. Trigonometric functions are introduced and developed to include inverses, general triangles and identities. Matrices provide an organizational structure in which to represent and solve complex problems. Students expand the concepts of complex numbers and the coordinate plane to represent and operate upon vectors. They apply methods from statistics to draw inferences and conclusions from data. Probability rounds out the course using counting methods, including their use in making and evaluating decisions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Unit 1: Students will use the unit circle to extend the domain of trigonometric functions to include all real numbers. Students will develop understanding of the radian measure of an angle, graph trigonometric functions, and derive and apply the Pythagorean identity.

Unit 2: Building on standards from Unit 1, students extend their study of the unit circle and trigonometric functions. Students will create inverses of trigonometric functions and use the inverse functions to solve trigonometric equations that arise in real-world problems.

Unit 3: Building on standards from Unit 1 and Unit 2, students will apply trigonometry to general triangles. Students will derive the trigonometric formula for the area of a triangle and prove and use the Laws of Sines and Cosines to solve problems.

Unit 4: Building on standards from the first three units, students will prove and use addition, subtraction, double, and half-angle formulas to solve problems.

Unit 5: Students will perform operations on matrices, use matrices in applications, and use matrices to represent and solve systems of equations.

Unit 6: Building on standards from previous courses, students will derive the equations of conic sections (parabolas, ellipses, and hyperbolas). Students will solve systems of a linear and quadratic equation in two variables.

Unit 7: Students will extend their understanding of complex numbers and their operations through graphical representations. Students will perform operations on vectors and use the operations to represent various quantities.

Unit 8: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Unit 9: Students will extend their study of probability by computing and interpreting probabilities of compound events. Students will calculate expected values and use them to solve problems and make informed decisions.

Mathematics | High School – Number and Quantity

Numbers and Number Systems: During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: they have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5(^{1/3})^3 = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities: In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

The Complex Number System — N.CN

Use properties of rational and irrational numbers.

MGSE9-12.N.CN.3 Find the conjugate of a complex number; use the conjugate to find the absolute value (modulus) and quotient of complex numbers.

Represent complex numbers and their operations on the complex plane.

MGSE9-12.N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

MGSE9-12.N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*

MGSE9-12.N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Vector and Matrix Quantities — N.VM

Represent and model with vector quantities.

MGSE9-12.N.VM.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

MGSE9-12.N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

MGSE9-12.N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

MGSE9-12.N.VM.4 Add and subtract vectors.

> **MGSE9-12.N.VM.4a** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

MGSE9-12.N.VM.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

MGSE9-12.N.VM.4c Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $(-\boldsymbol{w})$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

MGSE9-12.N.VM.5 Multiply a vector by a scalar.

MGSE9-12.N.VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

MGSE9-12.N.VM.5b Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

MGSE9-12.N.VM.6 Use matrices to represent and manipulate data, e.g., transformations of vectors.

MGSE9-12.N.VM.7 Multiply matrices by scalars to produce new matrices.

MGSE9-12.N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

MGSE9-12.N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

MGSE9-12.N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

MGSE9-12.N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

MGSE9-12.N.VM.12 Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Mathematics | High School – Algebra

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities: An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions: Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill.

Reasoning with Equations and Inequalities **A.REI**

Solve systems of equations

MGSE9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.

MGSE9-12.A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

MGSE9-12.A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Mathematics | High School – Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, and Coordinates:
Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs.

Interpreting Functions F.IF

Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

> **MGSE9-12.F.IF.7e** Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.

Building Functions F.BF

Build new functions from existing functions

MGSE9-12.F.BF.4 Find inverse functions.

> **MGSE9-12.F.BF.4d** Produce an invertible function from a non-invertible function by restricting the domain.

Trigonometric Functions **F.TF**

Extend the domain of trigonometric functions using the unit circle

MGSE9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

MGSE9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

MGSE9-12.F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

MGSE9-12.F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

MGSE9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

MGSE9-12.F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

MGSE9-12.F.TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

MGSE9-12.F.TF.8 Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.

MGSE9-12.F.TF.9 Prove addition, subtraction, double, and half-angle formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics | High School – Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations: The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Similarity, Right Triangles, and Trigonometry G.SRT

Apply trigonometry to general triangles

MGSE9-12.G.SRT.9 Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

MGSE9-12.G.SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.

MGSE9-12.G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations G.GPE

Translate between the geometric description and the equation for a conic section

MGSE9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.

MGSE9-12.G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Mathematics | High School—Statistics and Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.
Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions: Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Interpreting Categorical and Quantitative Data S.ID

Summarize, represent, and interpret data on a single count or measurement variable

MGSE9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, ~~mean absolute deviation~~, standard deviation) of two or more different data sets.

MGSE9-12.S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions S.IC

Understand and evaluate random processes underlying statistical experiments

MGSE9-12.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

MGSE9-12.S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

MGSE9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

MGSE9-12.S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

MGSE9-12.S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

MGSE9-12.S.IC.6 Evaluate reports based on data. *For example, determining quantitative or categorical data; collection methods; biases or flaws in data.*

Conditional Probability and the Rules of Probability S.CP

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MGSE9-12.S.CP.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)], and interpret the answer in terms of the model.

MGSE9-12.S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

Use Probability to Make Decisions S.CP

Calculate expected values and use them to solve problems

MGSE9-12.S.MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

MGSE9-12.S.MD.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

MGSE9-12.S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

MGSE9-12.S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

MGSE9-12.S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

> **MGSE9-12.S.MD.5a** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
>
> **MGSE9-12.S.MD.5b** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

MGSE9-12.S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

MGSE9-12.S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The BLUE links will provide access to classroom videos on each standard for mathematical practice accessed on the Inside Math website.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers

with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

More information of the Standards for Mathematical Standards may be found on the Inside Math website.

Connecting the Standards for Mathematical Practice to the Content Standards

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics. See Inside Math for more resources.

Classroom Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards based classroom and will support students' performances on the tasks in this unit and throughout the school year.

Strategies for Teaching and Learning

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
    - What level of support do my struggling students need in order to be successful with this unit?
    - In what way can I deepen the understanding of those students who are competent in this unit?
    - What real life connections can I make that will help my students utilize the skills practiced in this unit?

Tasks

The framework tasks represent the level of depth, rigor, and complexity expected of all Algebra I students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as performance tasks, they may also be used for teaching and learning (learning/scaffolding tasks). The table below provides a brief explanation of the types of tasks that teachers will find in the frameworks units for Algebra I.

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Short Cycle Task	Designed to exemplify the performance targets that the standards imply. The tasks, with the associated guidance, equip teachers to monitor overall progress in their students' mathematics.
Formative Assessment Lesson (FAL) ***more information on page 23**	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
3-Act Task ***more information on page 24**	A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
Achieve CCSS- CTE Classroom Tasks	Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

Formative Assessment Lessons (FALs)

The **Formative Assessment Lesson** is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to **inform** the instruction that will take place for the remainder of the unit.

Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

Videos of Georgia Teachers implementing FALs can be accessed HERE and a sample of a FAL lesson may be seen HERE

More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the Math Assessment Project's guide for teachers.

Formative Assessment Lessons can also be found at the following sites:
Mathematics Assessment Project
Kenton County Math Design Collaborative
MARS Tasks by grade level

A **sample FAL** with extensive dialog and suggestions for teachers may be found HERE. This resource will help teachers understand the flow and purpose of a FAL.

The **Math Assessment Project** has developed Professional Development Modules that are designed to help teachers with the practical and pedagogical challenges presented by these lessons.

Module 1 introduces the model of *formative assessment* used in the lessons, its theoretical background and practical implementation. Modules 2 & 3 look at the two types of *Classroom Challenges* in detail. Modules 4 & 5 explore two crucial pedagogical features of the lessons: asking probing questions and collaborative learning.

Georgia RESA's may be contacted about professional development on the use of FALs in the classroom. The request should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC).

Spotlight Tasks

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-Act Tasks

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

Guidelines for 3-Act Tasks and Patient Problem Solving (Teaching without the Textbook)
Adapted from Dan Meyer

Developing the mathematical Big Idea behind the 3-Act task:

- Create or find/use a clear visual which tells a brief, perplexing mathematical story. Video or live action works best. (See resource suggestions in the Guide to 3-Act Tasks)
- Video/visual should be real life and allow students to see the situation unfolding.
- Remove the initial literacy/mathematics concerns. Make as few language and/or math demands on students as possible. You are posing a mathematical question without words.
- The visual/video should inspire curiosity or perplexity which will be resolved via the mathematical big idea(s) used by students to answer their questions. You are creating an intellectual need or cognitive dissonance in students.

Enacting the 3-Act in the Classroom

Act 1 (The Question):

Set up student curiosity by sharing a scenario:

- Teacher says, "I'm going to show you something I came across and found interesting" or, "Watch this."
- Show video/visual.
- Teacher asks, "What do you notice/wonder?" and "What are the first questions that come to mind?"
- Students share observations/questions with a partner first, then with the class (Think-Pair-Share). Students have ownership of the questions because they posed them.

- Leave no student out of this questioning. Every student should have access to the scenario. No language or mathematical barriers. Low barrier to entry.
- Teacher records questions (on chart paper or digitally-visible to class) and ranks them by popularity.
- Determine which question(s) will be immediately pursued by the class. If you have a particular question in mind, and it isn't posed by students, you may have to do some skillful prompting to orient their question to serve the mathematical end. However, a good video should naturally lead to the question you hope they'll ask. You may wish to pilot your video on colleagues before showing it to students. If they don't ask the question you are after, your video may need some work.
- Teacher asks for estimated answers in response to the question(s). Ask first for best estimates, then request estimates which are too high and too low. Students are no defining and defending parameters for making sense of forthcoming answers.
- Teacher asks students to record their actual estimation for future reference.

Act 2 (Information Gathering):

Students gather information, draw on mathematical knowledge, understanding, and resources to answer the big question(s) from Act-1:

- Teacher asks, "What information do you need to answer our *main question*?"
- Students think of the important information they will need to answer their questions.
- Ask, "What mathematical tools do you have already at your disposal which would be useful in answering this question?"
- What mathematical tools might be useful which students don't already have? Help them develop those.
- Teacher offers smaller examples and asks probing questions.
    - What are you doing?
    - Why are you doing that?
    - What would happen if…?
    - Are you sure? How do you know?

Act 3 (The Reveal):

The payoff.

- Teacher shows the answer and validates students' solutions/answer.
- Teacher revisits estimates and determines closest estimate.
- Teacher compares techniques, and allows students to determine which is most efficient.

The Sequel:

- Students/teacher generalize the math to any case, and "algebrafy" the problem.
- Teacher poses an extension problem- best chance of student engagement if this extension connects to one of the many questions posed by students which were not the focus of Act 2, or is related to class discussion generated during Act 2.
- Teacher revisits or reintroduces student questions that were not addressed in Act 2.

Why Use 3-Act Tasks? A Teacher's Response

The short answer: It's what's best for kids!

If you want more, read on:

The need for students to make sense of problems can be addressed through tasks like these. The challenge for teachers is, to quote Dan Meyer, "be less helpful." (To clarify, being less helpful means to first allow students to generate questions they have about the picture or video they see in the first act, then give them information as they ask for it in act 2.) Less helpful does not mean give these tasks to students blindly, without support of any kind!

This entire process will likely cause some anxiety (for all). When jumping into 3-Act tasks for the first (second, third, . . .) time, students may not generate the suggested question. As a matter of fact, in this task about proportions and scale, students may ask many questions that are curious questions, but have nothing to do with the mathematics you want them to investigate. One question might be "How is that ball moving by itself?" It's important to record these and all other questions generated by students. This validates students' ideas. Over time, students will become accustomed to the routine of 3-act tasks and come to appreciate that there are certain kinds of mathematically answerable questions – most often related to quantity or measurement.

These kinds of tasks take time, practice, and patience. When presented with options to use problems like this with students, the easy thing for teachers to do is to set them aside for any number of "reasons." I've highlighted a few common "reasons" below with my commentary (in blue):

- This will take too long. I have a lot of content to cover. (Teaching students to think and reason is embedded in mathematical content at all levels - how can you **not** take this time?)
- They need to be taught the skills first, then maybe I'll try it. (An important part of learning mathematics lies in productive struggle and learning to persevere [SMP 1]. What better way to discern what students know and are able to do than with a mathematical context [problem] that lets them show you, based on the knowledge they already have - prior to any new information. To quote John Van de Walle, "Believe in kids and they will, flat out, amaze you!")
- My students can't do this. (Remember, whether you think they can or they can't, you're right!) (Also, this expectation of students persevering and solving problems is in every state's standards - and was there even before common core!)
- I'm giving up some control. (Yes, and this is a bit scary. You're empowering students to think and take charge of their learning. So, what can you do to make this *less* scary? **Do what we expect students to do:**
    - Persevere. Keep trying these and other open-beginning, -middle, and -ended problems. Take note of what's working and focus on it!

- Talk with a colleague (work with a partner). Find that critical friend at school, another school, online. . .
- Question (use #MTBoS on Twitter, or blogs, or Google: 3-act tasks).

The benefits of students learning to question, persevere, problem solve, and reason mathematically far outweigh any of the reasons (read excuses) above. The time spent up front, teaching through tasks such as these and other open problems, creates a huge pay-off later on. However, it is important to note, that the problems themselves are worth nothing without teachers setting the expectation that students: question, persevere, problem solve, and reason mathematically on a daily basis. Expecting these from students, and facilitating the training of how to do this consistently and with fidelity is principal to success for both students and teachers.

Yes, all of this takes time. For most of my classes, mid to late September (we start school at the beginning of August) is when students start to become comfortable with what problem solving really is. It's not word problems - mostly. It's not the problem set you do after the skill practice in the textbook. Problem solving is what you do when you don't know what to do! This is difficult to teach kids and it does take time. But it is worth it! More on this in a future blog!

Tips:

One strategy I've found that really helps students generate questions is to allow them to talk to their peers about what they notice and wonder first (Act 1). Students of all ages will be more likely to share once they have shared and tested their ideas with their peers. This does take time. As you do more of these types of problems, students will become familiar with the format and their comfort level may allow you to cut the amount of peer sharing time down before group sharing.

What do you do if they don't generate the question suggested? Well, there are several ways that this can be handled. If students generate a similar question, use it. Allowing students to struggle through their question and ask for information is one of the big ideas here. Sometimes, students realize that they may need to solve a different problem before they can actually find what they want. If students are way off, in their questions, teachers can direct students, carefully, by saying something like: "You all have generated some interesting questions. I'm not sure how many we can answer in this class. Do you think there's a question we could find that would allow us to use our knowledge of mathematics to find the answer to (insert quantity or measurement)?" Or, if they are really struggling, you can, again carefully, say "You know, I gave this problem to a class last year (or class, period, etc.) and they asked (insert something similar to the suggested question here). What do you think about that?" Be sure to allow students to share their thoughts.

After solving the main question, if there are other questions that have been generated by students, it's important to allow students to investigate these as well. Investigating these additional questions validates students' ideas and questions and builds a trusting, collaborative learning relationship between students and the teacher.

Overall, we're trying to help our students mathematize their world. We're best able to do that when we use situations that are relevant (no dog bandanas, please), engaging (create an intellectual need to know), and perplexing. If we continue to use textbook type problems that are too helpful, uninteresting, and let's face it, perplexing in all the wrong ways, we're not doing what's best for kids; we're training them to not be curious, not think, and worst of all . . . dislike math.

3-Act Task Resources:

- www.estimation180.com
- www.visualpatterns.org
- 101 Questions
- Dan Meyer's 3-Act Tasks
- Andrew Stadel
- Jenise Sexton
- Graham Fletcher
- Fawn Nguyen
- Robert Kaplinsky
- Open Middle
- Check out the Math Twitter Blog-o-Sphere (MTBoS) - you'll find tons of support and ideas!

Assessment Resources and Instructional Support Resources

The resource sites listed below are provided by the GADOE and are designed to support the instructional and assessment needs of teachers. All BLUE links will direct teachers to the site mentioned.

- Georgiastandards.org provides a gateway to a wealth of instructional links and information. Select the ELA/Math tab at the top to access specific math resources for GSE.

- MGSE Frameworks are "models of instruction" designed to support teachers in the implementation of the Georgia Standards of Excellence (GSE). The Georgia Department of Education, Office of Standards, Instruction, and Assessment has provided an example of the Curriculum Map for each grade level and examples of Frameworks aligned with the GSE to illustrate what can be implemented within the grade level. School systems and teachers are free to use these models as is; modify them to better serve classroom needs; or create their own curriculum maps, units and tasks. http://bit.ly/1AJddmx

- The Teacher Resource Link (TRL) is an application that delivers vetted and aligned digital resources to Georgia's teachers. TRL is accessible via the GADOE "tunnel" in conjunction with SLDS using the single sign-on process. The content is pushed to teachers based on course schedule.

- Georgia Virtual School content available on our Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.

- The Georgia Online Formative Assessment Resource (GOFAR) accessible through SLDS contains test items related to content areas assessed by the Georgia Milestones Assessment System and NAEP. Teachers and administrators can utilize the GOFAR to develop formative and summative assessments, aligned to the state-adopted content standards, to assist in informing daily instruction.

  The Georgia Online Formative Assessment Resource (GOFAR) provides the ability for Districts and Schools to assign benchmark and formative test items/tests to students in order to obtain information about student progress and instructional practice. GOFAR allows educators and their students to have access to a variety of test items – selected response and constructed response – that are aligned to the State-adopted content standards for Georgia's elementary, middle, and high schools.

  Students, staff, and classes are prepopulated and maintained through the State Longitudinal Data System (SLDS). Teachers and Administrators may view Exemplars and Rubrics in Item Preview. A scoring code may be distributed at a local level to help score constructed response items.

For GOFAR user guides and overview, please visit:
https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx

- Course/Grade Level WIKI spaces are available to post questions about a unit, a standard, the course, or any other GSE math related concern. Shared resources and information are also available at the site.

Internet Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GADOE does not endorse or recommend the purchase of or use of any particular resource.

General Resources

Illustrative Mathematics
Standards are illustrated with instructional and assessment tasks, lesson plans, and other curriculum resources.

Mathematics in Movies
Short movie clips related to a variety of math topics.

Mathematical Fiction
Plays, short stories, comic books and novels dealing with math.

The Shodor Educational Foundation
This website has extensive notes, lesson plans and applets aligned with the standards.

NEA Portal Arkansas Video Lessons on-line
The NEA portal has short videos aligned to each standard. This resource may be very helpful for students who need review at home.

Learnzillion
This is another good resource for parents and students who need a refresher on topics.

Math Words
This is a good reference for math terms.

National Library of Virtual Manipulatives
Java must be enabled for this applet to run. This website has a wealth of virtual manipulatives helpful for use in presentation. The resources are listed by domain.

Geogebra Download
Free software similar to Geometer's Sketchpad. This program has applications for algebra, geometry, and statistics.

Utah Resources
Open resource created by the Utah Education Network.

Resources for Problem-based Learning

Dan Meyer's Website
Dan Meyer has created many problem-based learning tasks. The tasks have great hooks for the students and are aligned to the standards in this spreadsheet.

Andrew Stadel
Andrew Stadel has created many problem-based learning tasks using the same format as Dan Meyer.

Robert Kaplinsky
Robert Kaplinsky has created many tasks that engage students with real life situations.

Geoff Krall's Emergent Math
Geoff Krall has created a curriculum map structured around problem-based learning tasks.